HERON FOUNTAIN AS A HYDRAULIC MACHINE

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A R T I C L E I N F O

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A B S T R A C T

In these days that many of resources on earth are draining , people are likely to hang on all the activities they have done before but nowadays we should endeavor for having better life without any harms on earth. One way is to use the equipment which can help us having enjoyable and desirable sweat life besides using resources in a rational way! In many years ago a physicist, Heron of Alexandria designed a fountain which worked without any external forces and just with three water containers and connecting tubes .The system can pump out the water by the inner forces and the pressure of the water in the system. The parameters which can affect on the water jet in this system have been studied in this research.

1 Introduction

Heron Fountain, designed by Heron of Alexandria, is a hydraulic machine that works with the differences of energies for its own construct. It contains three different containers that each of them contains air & water that pressure changes , can help the water moves in the system. Water is poured in the first container (the upper one is called Basin), moves to the second container (the lowest one, as the air and water supply), because of the energy differences . Then by moving the water downward, the air in the air supply moves to the third and last container (the middle one) that should have enough water for the water jet. So the duration of the jet depends on the amount of the water in this container because of that we call it water supply. The air coming from the air supply to the water supply, will press the water inside the water supply and by that pressure, the water jet goes out (Fig. 1).

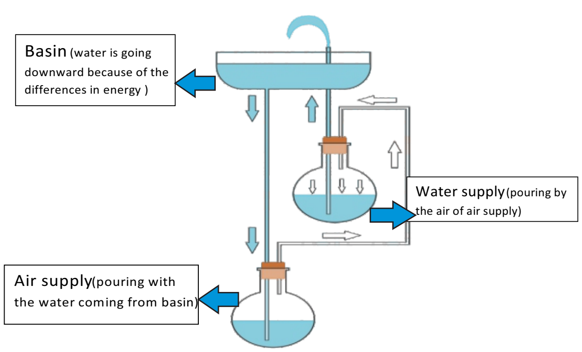


Fig. 1: Heron Fountain

2 Theories

The theories are used for this problem give us all the changes in energies or forces to predict what will happen in this system.

2-1 Bernoulli's Equation and Pascal's Law

In fluid dynamics, Bernoulli's principle states that an increase in the speed of a fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid's potential energy; although Bernoulli deduced that pressure decreases when the flow speed increases (Fig. 1). The principle is only applicable for : when the effects of irreversible processes (like turbulence) and non-adiabatic processes (e.g. heat radiation) are small and can be neglected. The simple form of Bernoulli's equation is valid for incompressible flows (e.g. most liquid flows and gases moving at low Mach number). More advanced forms may be applied to compressible flows at higher Mach numbers.

$$p_{atm} + \frac{\rho {v_1}^2}{2} + \rho g h_1 = p_{atm} + \frac{\rho {v_2}^2}{2} + \rho g h_2 \qquad (1)$$

To find the pressure differences in each two points pascal's law is used.

$$\Delta P = P_2 - P_1 = \rho g h_2 - \rho g h_1 = \rho g (h_2 - h_1) \qquad (2)$$

2-2 Flow rate and Bernoulli's Equation

In fluid dynamics, Bernoulli's principle states that an increase in the speed of a fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid's potential energy;[1,2]. Although Bernoulli deduced that pressure decreases when the flow speed increases [3,4]. The principle is only applicable for isentropic flows: when the effects of irreversible processes (like turbulence) and non-adiabatic processes (e.g. heat radiation) are small and can be neglected. The simple form of Bernoulli's equation is valid for incompressible flows (e.g. most liquid flows and gases moving at low Mach number). More advanced forms may be applied to compressible flows at higher Mach numbers.

In physics and engineering, in particular fluid dynamics the volume flow rate is the volume of fluid which passes per unit time; usually represented by the symbol Q (m^3/s) . The changes in volume flowing through the area would be zero for steady flow too [5].

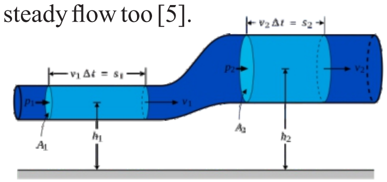


Fig. 2: Pascal's and Bernoulli's principles , flow rate comparison in different volumes

3 Materials & Methods

Here is a list of the materials have been used in our heron's fountain experiments:
- Two isolated plastic bottles as containers
- Isolated tubes for connecting different containers to each other for passing fluids (water& air)
- The plastic bowl used for the basin
- Stands for holding each container in different heights

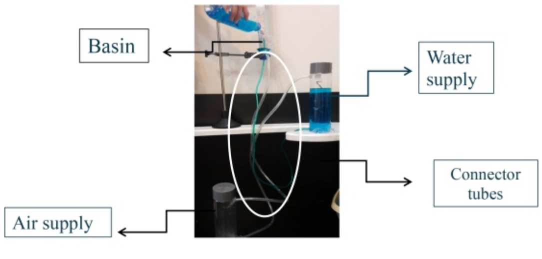


Fig. 3: Heron Fountain setup

At the first step, the water in the basin will comes down to the air supply because of the pressure differences based on the air pressure and water pressure:

$$p_{air} = p_0 + \rho g h_1 \quad (3)$$

After the downward movement of basin's water, we have some changes in the pressure of the air inside the water and air supplies. Based on the explained theories the air and water pressure in water supply can be calculated.

$$p_{water} = p_0 + \rho g h_2 \quad (4)$$

By transferring the air and increasing pressure inside the water supply, the water jet goes out.

$$\Delta p = p_{air} - p_{water} = \rho g(h_1 - h_2) \quad (5)$$

The height of the jet is found according to the flow rate and the pressure difference inside the tube that the water goes out from. Hagen-Poiseuille equation is used, which is about the incompressible and Newtonian fluid in laminar flow through a long cylindrical tube in a constant cross section A.

$$Q = \frac{\rho A^2}{8\pi\mu L}\Delta p \quad (6)$$

In this equation, μ is the viscosity of the fluid and L is the length of the tube . As we know the low viscosity and a wide-short pipe, are for turbulent flows so the Bernoulli's principle is used too.
then by Hagen's & Bernoulli's we got:

$$Q = \sqrt{2\rho}A\sqrt{\Delta P} \quad (7)$$

$$Q = c(\Delta P)^n \quad (8)$$

where c is constant and describes the fluid conductivity of the tube. c & n are given by experiments which n=1 in Hagen's and n=0.5 in Bernoulli's .
Now, from the equation for pressure differences and the flow rate , the flow rate inside the tube is given:

$$Q = c(\rho g(h_1 - h_2))^n \quad (9)$$

$$Q = \rho A v \quad (10)$$

$$v = \frac{c}{\rho A}(\rho g(h_1 - h_2))^n \quad (11)$$

Because of the conservation of energy in our system, the height of the fountain can be calculated by:

$$\frac{1}{2}mv^2 = mgH \rightarrow H = \frac{v^2}{2g} \quad (12)$$

Then the height of the fountain which depends on the conductivity of tube, c, is found . As the value of c is a complex subordinate of A , L , **ρ** , μ so the relations between H and these parameters cannot be found easily.

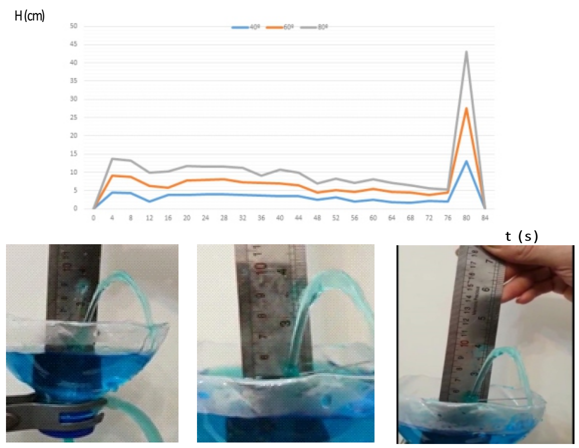
$$H = \frac{c^2 g^{2n-1}}{2\rho^{2-2n}A^2}(h_1 - h_2)^{2n} \quad (13)$$

4 Experiments& Results

Investigating the influence of some important parameters in our system shows:
- Changing the temperature of water will have some effects on the result of water jet.
- Changing the height between air & water supplies probably can change the pressure of the pumping water.
- Different diameters of tubes can cause some changes in the results because by changing the diameter, the existing pressure on water will change too.

4-1 Temperature

Changing the temperature of a liquid, will change the movements of the molecules too. Making temperature high, it will exacerbate the movements and lower temperature, will weaken the motion of molecules. These changes can affect on forces that are applied on the liquid.



40°C 60°C 80°C

Fig. 4: Height of water vs time in different temperatures

The starting peak is observed in these temperatures then the last peak which is upper than the first one is produced.
At the highest temperature (80°C) we see the highest peaks at first and last moments and higher duration; but, for two other temperatures are not the same; as the results show, as we lower the temp we'll get the lower range for height of fountain and exactly is the same for 40°C.

4-2 Different heights in air & water supplies

In upper height, high pressure probably can have higher fountain too.

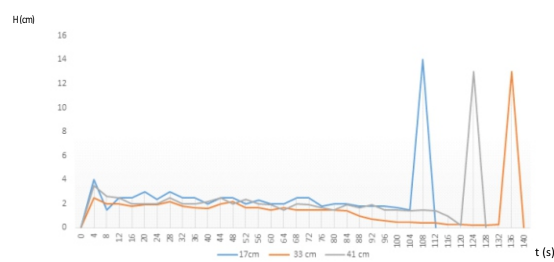


Fig. 5: Height of fountain vs time in different height

As the results from the experiments, higher distance give us higher range of fountain because it makes higher pressure for the water we have in the system. The higher height of fountain, is for higher height (41cm) and the lower one is for the lower height (17cm).

4-3 Different diameters

If we change the diameters of the last pipe that we are going to have the jet from, we'll see that the amount of the water coming out passing time is different in each diameter because the pressure is different in each pipe. Here different 3 diameters have been experimented with different results .

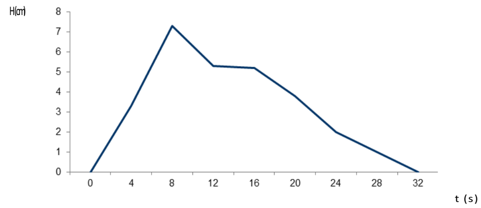


Fig. 6: 7.02 mm Diameter

As shown in Fig. (5), the first peak is in 8cm point, then we have the reduction of the height because of the energy loss and the amount of the water that is becoming low. But there is no the last peak as the other results because the pressure was not enough for pumping the water.

For 4.1mm diameter the first peak is seen but we noticed the point is lower than the last diameter but after the peak we have the duration but we don't have the last peak . The water doesn't have any movement on the pipe because there is no external forces on the system to pump the water out .

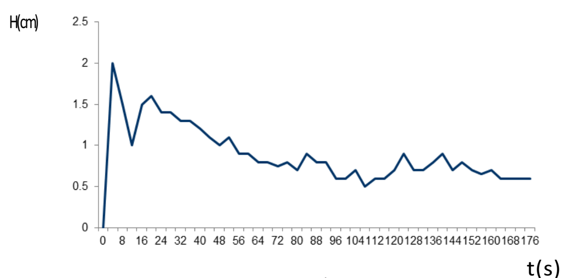


Fig. 7: 4.1mm Diameter

The best result of all diameters was for 3.2 mm that we can see the first and last peaks and the duration between and it shows that energy of the system is used in a correct way and energy loss was lower than the last two results.

Fig. 8: 3.2mm Diameter

5 Conclusion

1- As was explained in our experiments, in higher temperature of the water in the system, the pressure of the liquid will be higher too so we can have higher fountain.

2- As was mentioned , higher height will store more energy in the liquid and more energy equals to more pressure of water, and the more pressure causes more height for the water jet.

3- The changes in diameter of tubes will absolutely cause some effects on the height because the forces and energies in the system are high; as we lower the diameter of the waters pumping tube, it will need less energy for pumping it, but the high amount of forces let the water pumps higher.

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