THE RING OILER

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ABSTRACT

When a ring is put on a horizontal cylindrical shaft, it start to rotate around its axis at constant speed. It can travel along the shaft in either direction depending on the tilt of the ring. In this article we are going to investigate this phenomenon which was presented in IYPT 2018. We are going to investigate the phenomenon in a case were the ring is made of cardboard and the shaft is oiled. The reason behind the motion at the first place regarding to the microscopic point of view is explained that the motion of the shaft is intrinsic of the shaft and the rings circular shape. A mathematical model is used to clarify this phenomenon accurately.

1 Introduction

An oiled horizontal cylindrical shaft rotates around its axis at constant speed. Make a ring from a cardboard disc with the inner diameter roughly twice the diameter of the shaft and put the ring on the shaft. Depending on the tilt of the ring, it can travel along the shaft in either direction. This is IYPT 2018 problem which is investigated .

The first step is to explain why the ring moves along the shaft's axis in the first place. This is possible by taking a close microscopic look. Think of the ring at a set of particles, regarding the fact that both the ring and the cross section of the shaft are circular they only intersect in one point. However due to the imperfect shape of the shaft's surface and the ring's inner surface, this point extends to a small area (Fig. 1) which we call the contact area.

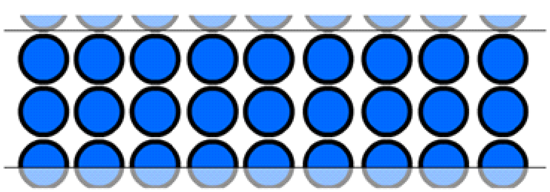
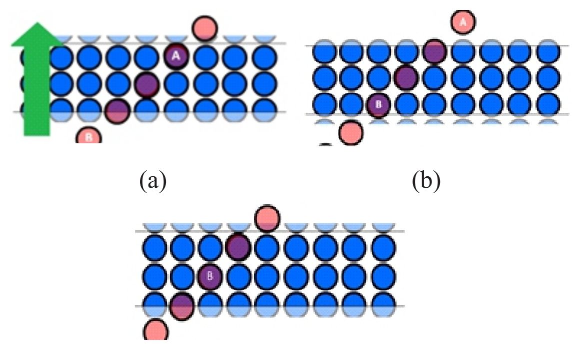


Fig . 1: the shaft can be thought of as a group of particles (shown in picture as circles). The ring and the shaft intersect in a small area called the contact point (the opaque area in between the two lines).

2 Theory and Methods

By looking at the particles of the ring as a set of particles too and a bit of simplifying we conclude that the particles of the ring tend to move in the direction towards which the particles of the shaft are moving as if they are “attached” to those particles by static friction. This kind of motion causes the particles of the ring, to repeatedly enter and exit the contact area. The new particle which enters the contact area is a bit to the side which is because of the tilt of the ring (Fig. 2) which perfectly explains why the ring moves to the side.

In figure (2a) the particles of the ring and the particles of the shaft move in the same direction and the same velocity due to the static friction between them. The two particles, A and B, are indicated. A, lays inside and B lays outside of the contact area. As shown in figure (2b) particle A is dragged outside and particle B is dragged into the contact area. Notice that particle B is a bit to the left due to the tilt of the ring. As explained in figure (2c) the transformation is endless due to the circular shapes of the ring and the cross section of the shaft and the points in contact (highlighted) keep changing. This causes the ring to move to the side.



(a) (b)

(c)

Fig. 2.a) The light colored circles represent the particles of the ring; b) particles A and B, They have both been displaced in the direction the shaft is rotating; c)The transformation is endless

Now let's have a look from the top. According to $\vec{v}$ ıat was said, the particles of the ring move with a velocity $\vec{v}$ in the direction the shaft is rotating. We can rewrite this vector as two other velocities. Let v_r be the velocity vector which is in the direction of the ring and v_x be the velocity vector in the direction of the shaft(Fig. 3).

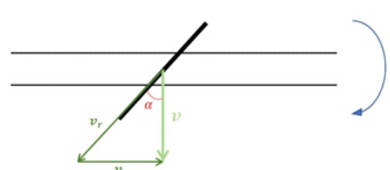


Fig. 3: The vectors of the velocity in rotating shaft

We know that the ring can't move in the direction of v_r hence this velocity only makes the ring revolve about its

own axis and v_x is the velocity of its linear motion.
From the above explanation we have [1]:

$$v = v_r.\cos\alpha \Rightarrow \omega_s.r = \omega_r R.\cos\alpha \Rightarrow \omega_s = \omega_r.\times \frac{R}{r}\cos\alpha \quad (1)$$

where ω_r and ω_s are the rotational velocities of the ring and the shaft and r and R are the radius of the shaft and the ring respectively.

As the problem lets us assume that $\frac{R}{r} = 2$ we could write:

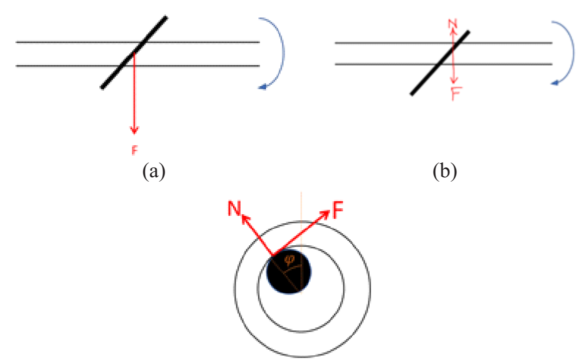
$$\omega_r = \frac{\omega_s}{2\cos\alpha} \quad (2)$$

$$v_x = v.\tan\alpha \Rightarrow v_x = \omega_s r.\tan\alpha \quad (3)$$

Here we can define the degrees of freedom of the rings motion which are:

v_x : Velocity of the rings linear motion
α : Angle between the rings orientation and the perpendicular line to the shafts length
ω_r : Rotational velocity of the rings revolving motion about its own axis.

Here, we assumed that the particles of the ring and particles of the shaft are attached and move with the same velocity called the critical velocity. However, we know that when the ring is released it doesn't have an initial velocity and slips along the shaft so it accelerates in the direction of the kinetic friction between them [2]. This force causes the ring to move in the direction of the force for a little time before the contact point moves slightly to the back of the ring causing the normal force to cancel out motion (Fig. 4 a/c).



(a) (b)

(c)

Fig. 4: a:) At the point when the ring is released on the shaft; b) After the ring in the direction of (F) the contact point moves a bit to the back of the ring giving a bit of angle to the normal force so that it cancels out the motion in the direction of F; c)from the side view

When the ring is released as shown in figure (4a) the kinetic friction force (F) gets applied to it, and the normal force is perpendicular to the viewing plane . From the figure (4c) it is clear how the ring has moved to the right and that the angle φ has appeared [3].

On from this point we use the force balance to approach the accelerating motion [3]. After that, we can break the friction force into two components, one of them –in the direction of the ring- causes the torque which makes the ring revolve around its own axis and the other, accelerates the ring. As the contact point has moved a bit to the back, the component of the normal force cancels out the perpendicular component of the remaining force component which later leaves us with a component of force, in the direction of the shaft which accelerates the ring until it reaches the limit velocity -v_x- (Fig. 5a/d).

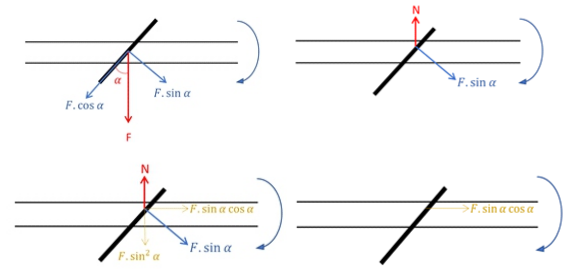


Fig. 5: a) breaking friction into two components one of which causes the torque which makes the ring revolve and the other causes the rings linear acceleration; b) due to the rings dislocation, the normal force has changed direction; c) dividing the remaining friction component into two new components, the perpendicular component must be canceled out by a component of the normal force as the ring is stabilized along that axis; d) one component of friction which accelerates the ring in the direction of the shaft.

Notice the direction of the friction force. The reason why a force in that direction could cause acceleration in the perpendicular direction is the interactions between the ring's linear and rotational movements the same way as the velocities did [2].

- $F.\sin\alpha\cos\alpha = ma$ (4)
- $Fr.\cos\alpha = I\dot{\omega}$ (5)
- $Fr.\sin\alpha\cos\alpha\sin\varphi = I\ddot{\alpha}$ (6)
- $F.\cos\alpha = mg.\sin\varphi$ (7)

By the equations we can calculate the third degree of freedom α .

The phenomenon could be divided into two main states:

1. Acceleration state: from the time when the ring is released until when it reaches the steady state. This is when the ring is accelerating.
2. Steady state: when the ring particles reach the velocity of the shaft and get attached to them.

During our experiments we sometimes observed that the ring changed directions randomly and started moving in the opposite direction without colliding to the side walls of the apparatus . We were surprised by this and succeeded to explain it by the changes of α .

Solving the differential equation (6) we can get the alpha per time plot and due to the fact that during the steady state, friction force disappears and regarding to Newton's first law of motion, the changes of alpha per time are linear in this state(we can draw the implicit plot of α per time like figure 6).

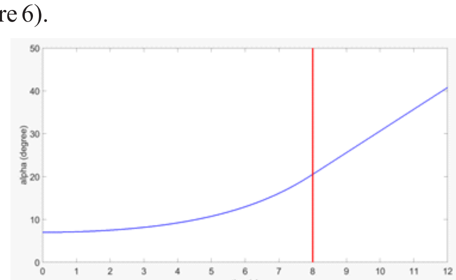


Fig. 6: the implicit alpha per time plot. Starting from the initial angle of 8 degrees and switching to the steady state after 8 seconds (the changes of alpha after that are linear)

This means that the angle between the ring and the shaft keeps decreasing over time. However this transformation cannot happen for ever. When the angle between the ring and the shaft gets lower than a specific value, the inner area of the ring hits the sides of the shaft and applies an impact. Reaction of this impact turns the ring in the opposite direction that causes this phenomenon which we call sudden rotation(Fig. 7).

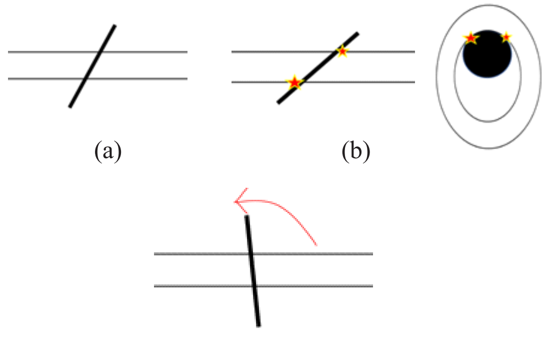


Fig. 7 : a) the ring has accelerated and entered the steady state. The angle between the ring and the shaft is decreasing constantly. b) the angle can only decrease down to a point and after that it hits the sides of the shaft. By taking a look through the shaft, we can understand this phenomenon easier (right side picture). At this point, a normal force gets applied to the ring at the indicated point which causes an impact. c) After the impact the force applied to the ring makes the ring to turn in the other direction, again with constant rotational velocity. Notice that the direction of the linear motion changes accordingly.

We continue by investigating the effect of different parameters. The effective and investigated parameters were [4]:

1. Side thickness of the ring
2. Air resistance
3. Oil
4. Material of the shaft

Which we are going to investigate here one by one.

2-1 Effect of the side thickness of the ring

We know that the surface of the ring and the shaft are imperfect and that our experiments have error. These errors are cause by many reasons. One of them is the bumpy and uneven surface of the ring. Mention the fact that the ring is cut from cardboard, cutting a perfect circle is close to impossible. Having this said one effect the side thickness has on the general function of the machine is increasing the random effects and errors in the run. The other effect is increasing the chance of sudden rotation because obviously, the minimum angle between a thin ring and the shaft is way less than the minimum angle between a thick ring and the shaft(Fig. 8).

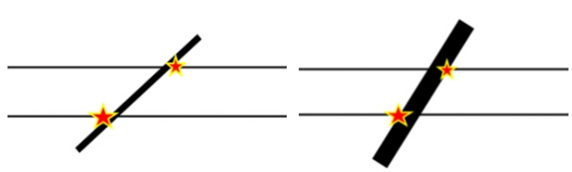


Fig. 8: Comparison between the minimum angle for a thick ring and a thin ring

2-2 Effect of air resistance

Due to the fact that the rings linear motion doesn't have a remarkably high velocity, the air colliding with the ring can smoothly steer along it. This causes the greatest amount of force to be applied to the top corner of the ring (Fig. 9). This force creates a torque and an acceleration backwards. Our experiments show that the effect of the torques is fairly negligible, whereas the acceleration, can lead to another phenomenon.

Suppose the ring is in steady state, the particles of the ring and the particles of the shaft are moving with the same velocity, however this external, air resistance force causes the particles of the ring to accelerate backwards and stay behind from the particles of the shaft. This puts the ring back into the accelerating state, causing the kinetic friction and most importantly, the angle φ keeps changing which proves our assumptions on the effect of air resistance.

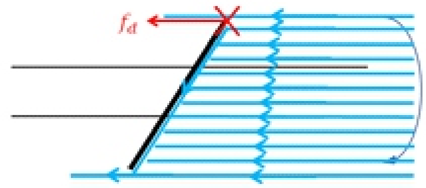


Fig. 9: the greatest amount of force to be applied to the top corner of the ring

2-3 Effect of Oil

We already know as a fact that the surfaces of the ring and the shaft are imperfect. The added lube can fit into the tiny microscopic gaps and decrease the error to a significant extent. This makes both the surfaces smoother thus deceases the kinetic friction coefficient and, as a result, the kinetic friction[1] .This makes the time that the ring spends in the accelerating state longer and omits the errors caused by great force. For example, it was observed that the ring would move forward too much and the angle φ becomes close to a right angle due to the high impact caused by the interior shape. The normal force in this case, applies an impact which makes the ring to bounce and detach from the ring for a moment which is considered as an error. The other effect of adding a lube is taking advantage of the surface tension. Stickiness of the oil causes the particles of the ring and the shaft to stick better to each other. In other words, we could say it increases the maximum static friction. This can limit the effects of environmental forces such as air friction or minor shakes of the setup. To prove this we have done some experiments without using oil, it is observed that the ring frequently bounced up and down on the shaft and we couldn't apply our theory because the ring and the shaft weren't attached anymore.

2-4 Effect of changing the material of the shaft

The main effect of changing the material means the changes of friction coefficient. These changes show themselves in changes of the angle φ .This comes from mathematical ratiocination below:

$$mg = N.\cos\varphi + F.\sin\varphi \tag{8}$$

$$F.\cos\alpha = mg.\sin\varphi \tag{9}$$

$$F = \mu.N$$

$$\mu = \frac{\tan\varphi}{\cos\alpha} \tag{10}$$

3 Experiments

Following the given instructions we constructed the apparatus. It consisted of an iron shaft resting between, two holes cut through a wooden body. The shaft was attached to a high speed electric motor which was connected to a speed controller and by that to a power source. This meant that we were able to alter the rotational velocity of the shaft by controlling the speed controller. As the question has asked us to, we cut rings out of a cardboard disc with a variety radiuses and side thicknesses. To reduce the error caused by the bumpy inner-surface of the ring we used a hot, circular piece of metal to burn out the extra parts and make the inner surface of the ring as ideal as possible. The setup was then placed on a reasonably leveled surface, measured by an electronic level. Two cameras recorded each run from the top view and the view through the shaft at the same time. We took the slow motion videos of the two cameras and used tracker to track the motion of the ring. The attributes we tracked were:

1- v_x , the linear velocity of the ring
2- ω_r ,the rotational velocity of the ring
3- α ,the complement of the angle between the ring and the shaft

And the variables of experimentation were:

1. The outer diameter of the ring
2. The side thickness of the ring
3. Presence or absence of oil
4. Length of the shaft

4 Results

4-1 Graphs and types of data

Five types of data are processed. The position , velocity, alpha and omega per time are shown in figures (10- 13) according to the parameters in table (1).

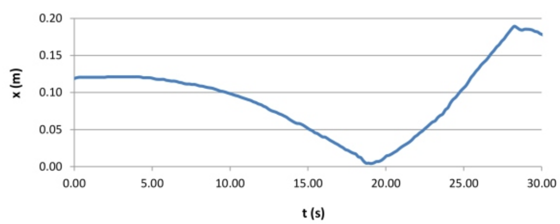


Fig. 10: Position versus time

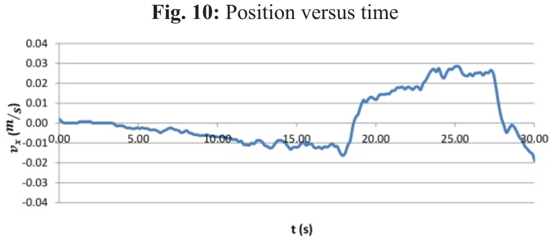


Fig. 11: Velocity versus time

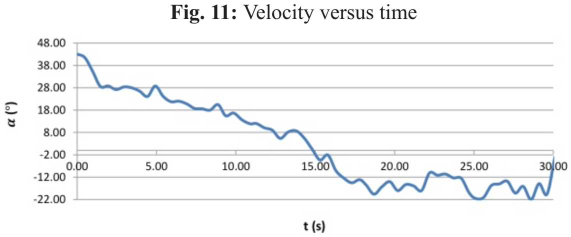


Fig. 12: Alpha versus time

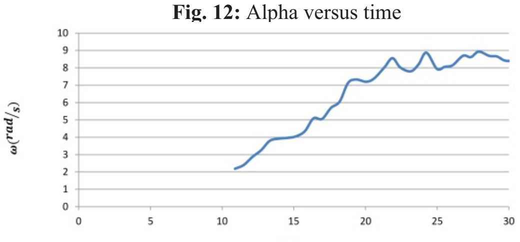


Fig. 13: Omega versus time

Table 1: Measured parameters in experimental setup

Shaft length	19cm
Mass of the ring	5 grams
Outer diameter of the ring	5 cm
Diameter of the shaft	0.9 cm

Figure (2) is taking the derivative of figure (1) but the rest of the graphs are direct data from tracker.

4-2 Theory precision:

The real values and results from theory are compared in figures (14 - 16).

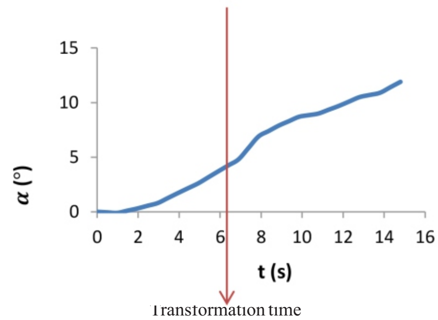
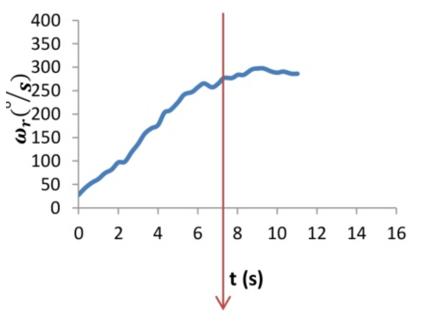
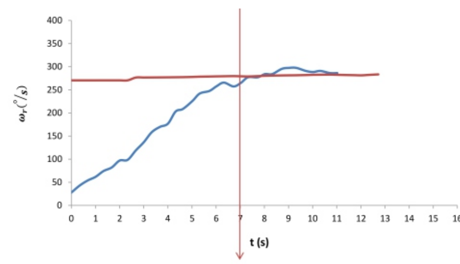


Fig. 14: The values of the alpha by Eq. (2) versus time



Transformation time

Fig. 15: The values of the omega by Eq. (2) versus time



Transformation time

Fig. 16: Comparison the values of the alpha by Eq. (2) and real values from experiments

As it is observed the trend line of the theory fits the actual data for the rotational velocity and this validates equation (1).

Transformation time is an implicitly chosen point to determine transformation from accelerating to rolling state. It is chosen by finding the first point after which omega becomes constant.

We take the values for alpha and omega from the two plots and calculate $\omega_r\ tg\ \alpha$ then plug it into equation (3) and compare the values (Fig. 17).

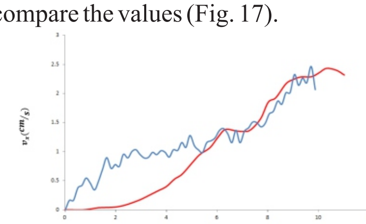


Fig. 17: Comparison yield that the theory perfectly predicts the velocity trend. Equations are supposed to work in the rolling state.

Equation (6) is a differential equation. We tried to solve this but this equation doesn't seem to have an explicit solution. So computer algorithm was used to draw the explicit plot of alpha per time for each specific case. Figure (18) is an example.

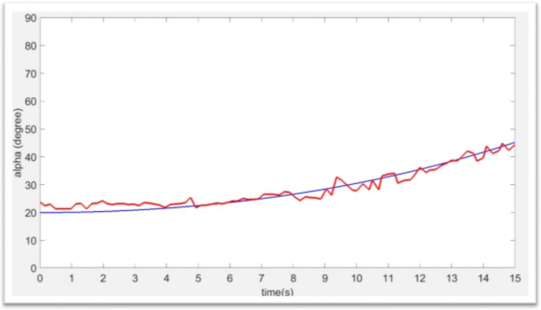


Fig. 18: Again the theory matches the experimental data and this validates equation (6)

5 Conclusion

The reasons, and the frequently observed cases were investigated in this phenomenon .It is concluded that the horizontal movement of the ring is intrinsic of the circular shapes and the angle between the ring and the shaft. This problem is dividable into two states, the Rolling and Accelerating states. The ring has three Degrees of freedom: Alpha (α) , Omega (ω) and linear velocity (v_x). Each of these attributes can be calculated by the equations (6), (3) and (2), respectively. The more the side thickness of the ring, The more the error. The better the quality of lubrication, the smoother, better and more accurate our results. Air resistance doesn't have a significant effect on the movement, it just causes looping through rolling and accelerating states.

References

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